THE TRIANGLE AS AN AID TO DISCOVERY.

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PART II.

WITH Barrow, and his disciple Newton, the triangle enters on a new career of usefulness. From Alexandrian times the idea of movement had been rigorously excluded from Geometry. Ignoring the fact that all our knowledge is derived from the movement of voluntary muscle, the Greeks devoted themselves to a Geometry of Contemplation, not of action. Almost the only lapse from this stern ideal is the concession which allows you to move a triangle and place it upon another in order to prove the equality of the sides. You are not allowed to contemplate it as rotating, in which case you might prove Euclid I. 5 in half a dozen lines; or to imagine it as growing by the upward extension of the apex, in which case the doctrine of parallels and the size of the interior angles could have been established with fewer difficulties than Euclid has encountered. If a line be regarded as the path of a moving point, if an angle be regarded as the inclination of a rotating line, if parallel lines be regarded as the sides of a triangle whose apex has been removed to infinity, then the scope of geometry is immensely extended and a new and powerful instrument of research is created. That was the lesson that Barrow taught to Newton, and the result was the *Principia,* a work which exhibits the order of nature as deductions from the movements of the triangle.

There is one incidental conception of Barrow's which had an immediate effect on the future of mathematics and has for that reason been too consistently regarded as purely mathematical and in no way related to the general logic of nature. That is the conception of orders of magnitude. In nature there is nothing great or small, but every magnitude may be conceived as negligibly small in comparison with some other magnitude and, in its turn, there is some third magnitude as far below the second as the second is below the first. Suppose a triangle extended in height so that the vertical angle is very small in comparison with either of the angles at the base, then the difference in length between the two sides is indefinitely small in comparison with the length of either of them. If then the original triangle shrinks towards a microscopic size, the difference between the two sides becomes an infinitesimal of the second order and may be disregarded in further investigations. Nothing is to be conceived so small but it may be indefinitely larger than something else: nothing so large but it may be evanescent in comparison with something else. That our minds find some difficulty in accommodating ourselves to this principle arises from the fact that we are accustomed to regard the movement of our own limbs as setting a standard of size. It is as though we occupied a midway position in the universe and it is not altogether easy to recognize that the ten-millionth part of an inch contains as much room for internal complexity as the solar system. But the course of investigation requires of us, now more than ever, that we should recognize the smallest particle we are called upon to consider as a system with internal problems of its own comparable in difficulty with those of stellar astronomy.

Up to the end of the seventeenth century science had developed by geometrical methods alone, but the *Vis Viva* controversy marks a parting of the ways. The universe of Galileo and Newton is a finished work endowed once and for all with a certain quantity of momentum by a Creator "who in an infinite space as in its sensorium sees, discerns and understands everything" (Newton, *Opticks*). Leibnitz suggested a different sort of universe. A man of restless intellect, his universe is restless too; its ultimate realities are active elemental energies who create their own universe as they go along. He challenges the Newtonian (or Cartesian) concept of momentum and declares that the proper measure of action is the square of the velocity, not the simple unit. He endeavours to convict the Newtonians of error by pointing out that though a body only doubles its velocity in falling first from a height of one foot and afterwards from a height of four feet, whereas it requires only the same effort to raise a pound four feet as to raise four pounds one foot high. When Clarke answers that time is an element which has to be taken into account in the raising or falling of a body, Leibnitz replies that time ought to be left out of account for Force exists in itself.

The remarkable feature of the controversy was that whenever particular instances were quoted both controversialists arrived at the same numerical results and yet they never arrived at an agreement in principle. The controversy tended to degenerate into a mere dispute about words, as though one should contend that he heard the sound of the wind in the trees and another that what he really heard was the sound of the trees in the wind. If Leibnitz, who called in the aid of the triangle to discover his series for $\frac{\pi}{4}$, had only invoked the triangle in the *vis viva* dilemma he might have reconciled the controversialists. The triangles constructed by Galileo on the arms of the lever, or unequal-armed balance, show the masses and the spaces covered in unit-time; they measure the momenta. Triangles constructed by the arms themselves in motion may be used, and if you multiply the weight by the circular measure of the arc through which it moves you arrive at the duplicate ratio or *vis viva*.

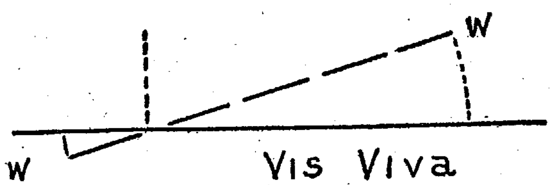

FIG 9

In the one case you are concerned with the problem of supporting a weight; in the other case with the problem of carrying it upstairs. The two results are perfectly consistent and they were ultimately resolved into a more general statement by D'Alembert.

Up to this point in its history geometrical method had been supreme. There was no physical science which was not geometrical. It is not equally so in the later stages. During the great period of French analysis the mathematicians were always spoken of as geometers, although they never thought of using a diagram and appeared to dispense entirely with every consideration of shape or size. Lazare Carnot wrote his "Metaphysic of the Infinitesimal Calculus" to show that the whole of the higher mathematics is geometrical in origin and that it becomes illogical when divested of geometrical meaning. In the hands of his son, Sadi Carnot, Heat became a geometrical science. Optics had always been such, and Chemistry endeavoured to become such in the mind of Dalton, of Prout, and of Van't Hoff. Rowan Hamilton's Principle of Varying Action,—

which led to his reduction of six dynamical equations to two, and which enabled him to predict that a ray of light would form a hollow cone within a crystal of arragonite and a hollow cylinder on emerging—was founded entirely on geometrical considerations. He desired a general expression for the surface formed by all possible rays (or lines of attraction) diverging from a point. If he had lived to give a more systematic form to his doctrine of varying action he would probably have given to the principle of Relativity a more logical form than it has yet attained. It was the Hamiltonian equations, combined with Faraday's purely geometrical conception of the magnetic field, which enabled Maxwell to predict the existence of those special waves which Hertz, with his triangle of pitch, realised and which Marconi harnessed for the service of humanity.

Are geometrical methods ever again to be supreme in Physical Science? Is there any reason in the nature of things why they should be?

Consider the problems presented by the three states, gaseous, liquid and solid. Of the gaseous state we have a complete geometrical plan or chart representing it as a system of particles in constant motion. Their paths, their collisions, their sizes, even their shapes and masses can be enumerated: their pressure on the envelope in which they are contained can be shown to vary as the square of the mean velocity of the molecules. Here we have a complete working model of a gas and the only concept of a non-geometrical nature which enters into it—that of pressure—enters as a mere subordinate detail of calculation, and not as any essential part of the fabric. Of the liquid state we have no such chart: we possess indeed means of calculating its behaviour under certain conditions, but we possess no plan or chart of the liquid state in general. Of the solid state we know nothing at all; not even the scaffolding has yet been erected, still less the fabric.

Or take the case of Chemistry. Here we have a Science which in the rate of accumulation of details has surpassed all others; and yet at the present moment we have no means of defining what is meant by an element. Is not the reason to be sought in the fact that from the days of Prout to those of Pasteur, Chemists rigidly set their faces against any attempt at a geometrical treatment of Chemical Theory. When Pasteur's hint as to the arrangement of the molecules in certain crystals was taken up by Van't Hoff and expanded into the tetrahedral plan of the carbon compound molecule, how did the German professors receive the new theory. Thus wrote

Kolbe: "It is one of the signs of the times that "modern chemists hold themselves bound and consider themselves to be in a position to give an explanation of everything, and when their knowledge fails them, to make use of supernatural explanations. . . . And herewith he makes it clear that he has gone over from the camp of the true investigators to that of the speculative philosophers of ominous memory." This diatribe was issued in 1878 and before Kolbe's death the geometrical conception of the carbon molecule had justified itself by predicting the optical activities of compounds unknown at the time of its publication. The future of Chemical Science is now, however, in the hands of the physicists who are fully alive to the desirability of constructing a geometrical model of the molecule, and a time-chart of its movements, external and internal.

That phrase "Time Chart" expresses as nearly as possible the aim of scientific investigation, in the attempt to construct an Order of Nature out of the elemental chaos with which crude Nature confronts us. The first step is to draw a map and the second step to show how this map alters in time. The result bears much the same relation to the crude reality as a chart of the Bay of Biscay bears to the bay itself. But it answers its purpose, for its soundings expressed in black ink on blue paper enable the mariner to navigate the sea in safety.

If the construction of a Time-Chart is the main object of Science, why is not one method as good as another? Why should a system founded on shapes and sizes be preferable to a system founded on energies, intensities, turbulence and tumult, as the ultimate verities? Do we not need both? And the answer would seem to be that we need shapes and sizes first and afterwards the means of measuring, describing and comparing them. The former produce a framework clear, rigorous and completely intelligible; the latter produce a filling-in which is incomplete, vague and partially unintelligible. And the reason lies in the vehicle of language through which description and calculation must be conveyed. The investigating element may be purely symbolic consisting of partial differential equations, but these have to be translated into sentences and in the process they tend to become incoherent.

The fact is that language is not altogether adequate to such enterprises as Physical Science is engaged in. Conception is always in advance of articulation and our vocabulary is too limited for our requirements. At the present moment the word "mass" has to do duty for three or four concepts which ought to be sharply distinguished, and physicists have recently deplored the difficulty they experience in defining such an apparently simple idea as that of Density.

The meaning of a general word in common use may be compared with the image in a binocular out of focus; and the occasions on which it can be brought to a clear image free from fringes and ambiguities are comparatively few. Practically they may be said to be limited to the occasions when we are dealing with shapes and sizes, and for that reason, and for no more transcendental reason, geometrical statements occupy a preeminent position of certitude. The triangle once defined is always the triangle and never anything else. That is the key to Kant's *Prolegomena.*

Of words representing activities, or intensities, there is not one which is free from the suspicion of ambiguity and therefore not one which is completely intelligible. Newton himself felt this defect in his doctrine of attraction and would fain have made it purely geometrical, dispensing with such words as force and impulse. What would he have thought of the equations in which Oswald identifies a gram of ponderable matter with a pressure multiplied by the square of a velocity? Such an equation may be romantic, seething with possibilities, but as the description of a particle it is imperfectly satisfying.

It is yet possible that physics, having been for sometime mainly occupied with energy factors, may bring them into subordination again to geometrical ideals. If so it must of necessity liberate itself from the habit of regarding spatial and temporal characters as on the same plane of importance. Duration is something far more fundamental than space. A geometry which has adopted the idea of motion has necessarily to adopt the ideas of beginning, of continuity, and of ending. Space is something which has happened in the course of time. If the physicist prefers for convenience of calculation to treat time as the fourth member of a space equation, he must nevertheless allow that his device is only a convention necessary for the purposes of physical bookkeeping in order to enable him to produce a trial balance sheet of the universe. For the historian too has his time, and so has the geologist; and either those eras are to be conceived as portions of the duration in which the earth's movement is being accomplished, or else in the language of Jowetts' satirist "There is no knowledge."